Parallel Simulated Annealing with Adaptive Temperature Determined by Genetic Algorithm

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Abstract— **Simulated annealing (SA) is an effective general heuristic method for solving many combinatorial optimization problems. This paper deals with the two problems in SA. One is the long computational time of the numerical annealings, and the solution to it is the parallel processing of SA. The other one is the determination of the appropriate temperature schedule in SA, and the solution to it is the introduction of an adaptive mechanism for changing the temperature. The multiple SA processes are performed in multiple processors, and the temperatures in the SA processes are determined by a genetic algorithms. The proposed method is applied to solve many TSPs (Traveling Salesman Problems), and it is found that the method is very effective and usefule.**

Keywords— **Simulated Annealing, Genetic Algorithm, Temperature, Traveling Salesman Problem.**

I. Introduction

There is a strong incentive to parallelize the computation for optimization problems since it requires many iterations of analysis. Especially, simulated annealing, which are very effective for solving complicated optimization problems with many optima, requires tremendous computational power. Consequently, parallelization of the method is very important.

It was Kirkpatrick et al. who first proposed simulated annealing, SA, as a method for solving combinatorial optimization problems[1]. It is reported that SA is very useful for several types of combinatorial optimization problems. However, the most remarkable disadvantages are that it needs a lot of time to find the optimum solution and it is very difficult to determine the proper cooling schedule.

Because of the progress of parallel computers, there are several studies on SA using parallel computers. Among these studies, the temperature parallel simulated annealing (TPSA), which was called the time-homogenous parallel annealing[2] before, is one of the algorithms that can overcome the cooling schedule problem, and that can reduce the computation time also.

However, the higher temperatures assigned to some of the processors of a parallel computer can be considered to be too high as the annealing proceeds since the annealings at the higher temperature do not yield the convergence of solutions. Therefore, the effectiveness of multiple processors is reduced in TPSA.

In order to overcome this problem, we propose a new method for determining the temperature adaptively as the multiple annealings proceed. The temperatures assigned to all the processors of a parallel computer are determined by a genetic algorithm(GA)[3]. The temperatures are dynamically changed to appropriate values during the annealing process.

II. Important Temperature Region for TSP

A. Important Temperature Region

There is an important temperature region in a temperature schedule of SA, where the search is carried out very efficiently. Harry[4] found that a specific constant temperature in SA yielded good solutions for TSPs, and Mark[5] found that the similar results for quadratic assignment problems.

Such specific constant temperatures are called the important temperature regions in SA in this paper, and our proposed method is constructed with the important temperature region.

B. Traveling Salesman Problem

The traveling salesman problem (TSP) used in this paper is a problem for finding the minimum distance of a tour of visiting all the finite number of cities and returning to the starting point.

The tour distance is expressed as follows[6]:

$$\sum_{i=1}^{N-1} d(v_\pi(i), v_\pi(i+1)) + d(v_\pi(N), v_\pi(1)) \qquad (1)$$

where $v(i)$ is the i-th point (city) in a tour π, $d(v(i), v(j))$ is the distance between two points, and $d(v(i), v(j)) = d(v(j), v(i))$.

The neighborhood structure used in this paper is the 2-change neighborhood[6], which is the most fundamental one for TSPs. The problems used for evaluating the proposed method are 14 problems which are selected arbitrary from TSPLIB[7].

C. Confirmation of the Existence of the Important Temperature Region

The important temperature region for each TSP is found by performing many SAs with various constant temperatures and comparing the qualities of the solutions obtained.

The 32 temperatures used are determined by the following procedure, which is commonly used for determining temperatures in TPSA[6].

1) Determine the maximum temperature, where the worst transition is accepted with the probability of 50%. The worst transition is determined by some preliminary experiments.

2) Determine the minimum temperature, where a bad transition is accepted only once for a prescribed annealing steps (20 times the number of cities).

3) Divide geometrically the range between the maximum and minimum temperatures into 32 temperatures.

In order to find the important temperature region, typical five TSPs (traveling salesman problem) from TSPLIB[7] were solved by SA with various constant temperatures.

The one of the experimental results is shown in Fig. 1, where the tour distances of eil51, which is one of the TSPs used, are shown as a function of constant annealing temperature. The values shown are the average of 20 trials.

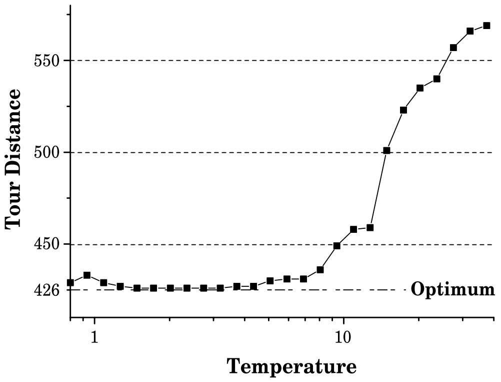

Fig. 1. Result of SA with constant temperature for eil51

From Fig. 1, nearly optimum solutions are found between 1.5 to 3, and this region can be considered as the important temperature region for the eil51 problem. Similar results are obtained for the other TSPs, and the detail of the experimental results are summarized in Table I, where T_{opt} means the important temperature region, the error ratio means the difference between the exact and obtained solutions divided by the exact solution (the global optimum), and gOptimumh means the obtained tour distance averaged over 20 trials.

It is found that all TSPs have respective important temperature regions, and a SA with such important temperature region can provide a very good performance.

D. Characteristics of the Transition of the Solution

The transitions of the solution in the important temperature region in SA is investigated in order to find the mechanism for providing good solutions. The histories of the transitions of the solution for eil51 are shown in Fig. 2, where the three histories of the tour distance for three temperatures are shown as a function of the annealing steps. One temperature is the important one, one is above the important temperature, and the other is below the important temperature.

TABLE I
Important temperatures for several TSPs.

Problem	T_{opt}	Error ratio	Optimum
a280	2-4	4.36E-3	2579
berlin52	26-45	0.00E+0	7542
bier127	150-250	3.43E-3	118282
ch130	10-15	4.72E-3	6110
ch150	8-12	2.24E-3	6528
eil51	1.5-3	0.00E+0	426
eil101	1.3-2	2.12E-4	629
gil262	1-3	8.10E-3	2378
kroA100	45-80	4.17E-4	21282
lin105	35-55	6.10E-4	14379
lin318	22-27	1.68E-2	42029
pr76	300-400	6.29E-5	108159
pr152	80-120	1.36E-3	73682
tsp225	3-5	2.65E-2	3916

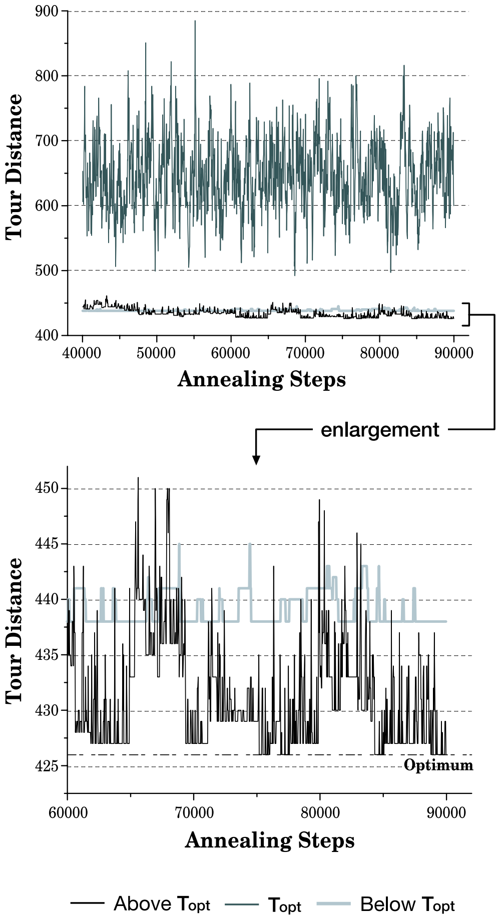

Fig. 2. Transitions of solutions at different temperatures for eil51.

It can be found that the annealing at the temperature above the important temperature shows a large fluctuation, but the solution does not reach the optimum solution. On the other hand, the annealing at the temperature below the important temperature shows relatively good solution, but the fluctuations are very little. Therefore, the possibility of finding the optimum solution is very low. Compared with these results, the annealing at the important temperature shows a large fluctuation and a low average. Therefore, even the annealing at constant important temperature can provide a very near optimum solution.

The characteristics of the transition of the solution at the important temperature are as follows:

1) Relatively large fluctuations
2) Relatively good solution

Using these characteristics, a new method for determining the important temperature can be constructed and a new adaptive SA can be devised based on the important temperature.

III. Parallel Simulated Annealing with Adaptive Temperature

A. Concept of PSA/AT

We found the important temperature region in SA, but such temperature range is problem-dependent and it is difficult to find during the search. Therefore, we consider an adaptive mechanism for determining the important temperature for parallel SA (PSA). This method is called the parallel simulated annealing with adaptive temperature (PSA/AT). Each temperature of PSA is determined by a conventional GA (genetic algorithm). The schematic of the proposed method is shown in Fig. 3. It should be noted that the temperature can be evolved since multiple SAs are performed in parallel, and the population of the temperatures can be constructed.

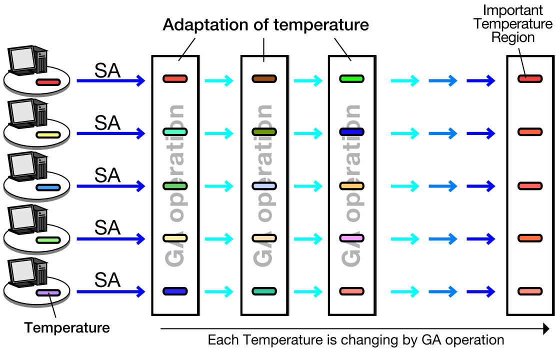

Fig. 3. Schematic of the parallel SA with adaptive temperatures

B. Algorithm of PSA/AT

Figure 4 shows the algorithm of PSA/AT, where the initial temperatures are generated with random numbers and multiple SAs starts with these temperatures. After the prescribed annealing steps the temperatures are evolved by using GA operators, and new temperatures are assigned to the multiple SAs.

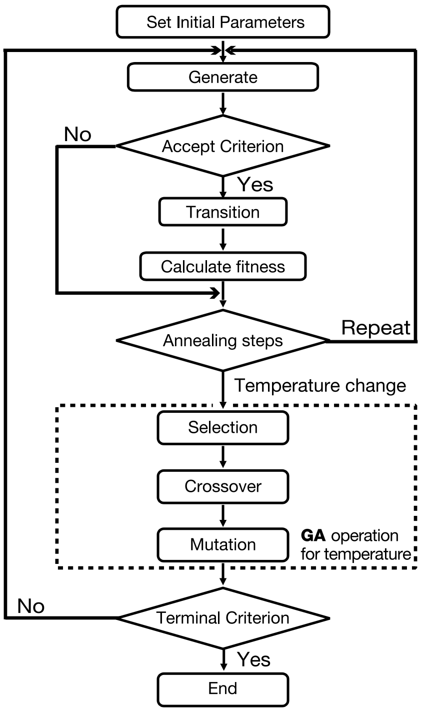

Fig. 4. Algorithm of PSA/AT

The features of the method is as follows.

1) Generate , Accept Criterion , Transition

The generation of a new solution, the accept criterion, and the transition of the solution are the same as conventional SA. These procedures are the same as the conventional SA.

2) Temperature is determined by GA

The synchronization of all the processes is done with a certain period, and the temperatures are determined by a GA based on the fitness value calculated from the energy of the solutions.

The temperatures of the multiple SA processes are changed by crossover, mutation, and selection. That is, the temperature that gives good solutions survives. Thus, all the temperatures are expected to be converged to the important temperature region.

C. Fitness value

The fitness value for the selection of better temperatures are defined based on the characteristics mentioned in section B, as follows.

$$Fitness = \sum (Baseline - Enregy) \tag{2}$$

That is, the fitness is the summation of the differential energy value lowered below a baseline which is an average of the energies of all SA processes performed in parallel. Fig. 5 illustrates the evaluation of the fitness. With this fitness value, the temperature with the characteristics of the transitions at the important temperature can be selected as a good temperature.

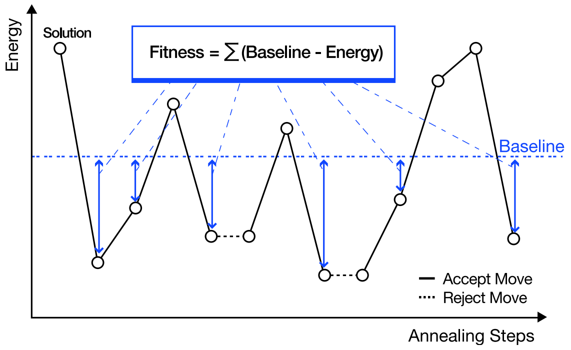

Fig. 5. Calculate fitness value

D. *Coding of temperature*

The temperature in SA is a very important parameter and the performance of a SA is heavily dependent on an appropriate temperature schedule[8]. In practice, the exponential cooling schedule is widely used. Therefore, the temperature in PSA/AT is expressed by Eq. (4).

$$T = \exp X \tag{3}$$

where T is the temperature, and X is a binary coded number. Therefore, the range of temperature can be very wide. The crossover used is one-point crossover.

IV. NUMERICAL EXPERIMENTS

A. *Outline of experiments*

To verify the validity and the effectiveness of the proposed method, PSA/AT, TPSA and the sequential SA are applied to solve 14 TSPs.

The parameters used are shown in Table II. The number of temperatures, which is the same as the number of SA processes, and the total anealing steps are the same as in Ref. [6]. The maximum and minimum temperatures are determined so as to include the important temperature region in this range surely.

TABLE II
PARAMETERS USED FOR PSA/AT

Num of SA processes	32
Maximum Temperature	10000
Minimum Temperature	0.01
Temperature change interval	20n
Total steps	20n×160
bit length	10
Selection method	Roulette
Crossover rate	0.01
Mutation rate	0.1

n : number of cities

For the TPSA, the temperatures are determined by the conventional method mentioned in section II-D, and the interval for exchange solutions are the same as the temperature change interval shown in Table II.

B. *Parallel computer used*

The parallel computer used is a PC cluster (Cambria cluster system) with 256 processor elements shown in Fig. 6, and 32 nodes are used for the experiments. The detail of the computer system is shown in Table III.

Fig. 6. PC cluster used

TABLE III
DETAIL OF THE PC CLUSTER USED

CPU	Pentium3 800MHz(256CPU)
Memory	256MB×256
Network	FastEthernet
OS	Debian GNU/Linux 2.4
Communication	mpich

C. *Experiment Result*

Tables IV and V show the averages and medians of the error ratio of the solutions obtained over 30 trials. All values are related to the best solutions in each trial. From these tables, it is found that the error ratios are very small compared to TPSA and the sequential SA(SSA), and the proposed method, PSA/AT, shows very excellent performance.

TABLE IV
ERROR RATIOS OF THE SOLUTIONS OBTAINED (AVERAGE)

Problem	PSA/AT	TPSA	SSA
a280	0	1.41E-03	4.82E-03
berlin52	0	0	0
bier127	1.61E-05	1.51E-03	1.00E-02
ch130	1.47E-04	2.01E-03	6.49E-03
ch150	5.36E-04	1.81E-03	5.26E-03
eil51	0	0	1.49E-03
eil101	0	0	4.98E-03
gil262	1.23E-3	5.80E-03	5.56E-03
kroA100	0	1.97E-04	3.89E-03
lin105	0	0	3.23E-03
lin318	5.07E-03	1.30E-02	1.29E-02
pr76	0	2.43E-05	2.71E-03
pr152	2.48E-04	1.06E-03	6.14E-03
tsp225	1.23E-03	7.01E-03	1.46E-02

Table VI shows the ratio of finding the optimum, that is, 0.4 means the number of trials finding the optimum

TABLE V

Error ratios of the solutions obtained (Median)

Problem	PSA/AT	TPSA	SSA
a280	0	7.75E-04	1.94E-03
berlin52	0	0	0
bier127	0	1.33E-03	1.08E-02
ch130	0	2.29E-03	3.93E-03
ch150	0	1.84E-03	5.06E-03
eil51	0	0	2.35E-03
eil101	0	0	3.18E-03
gil262	1.26E-03	5.05E-03	4.63E-03
kroA100	0	0	3.81E-03
lin105	0	0	0
lin318	5.31E-03	1.41E-02	1.24E-02
pr76	0	0	1.12E-03
pr152	0	1.38E-03	5.52E-03
tsp225	7.66E-04	6.13E-03	1.53E-02

is 12 out of 30 trials. From this table, it is concluded that the proposed method shows very high reliability in finding the optimum. Consequently, PSA/AT is found to be vary effective and useful method.

TABLE VI

Ratios of finding the optimum solutions

Problem	PSA/AT	TPSA	SSA
a280	1.00	0.40	0.23
berlin52	1.00	1.00	1.00
bier127	0.93	0.30	0.07
ch130	0.90	0.13	0.13
ch150	0.53	0.17	0.03
eil51	1.00	1.00	0.37
eil101	1.00	1.00	0.10
gil262	0.03	0.00	0.00
kroA100	1.00	0.77	0.03
lin105	1.00	1.00	0.57
lin318	0.00	0.00	0.00
pr76	1.00	0.97	0.17
pr152	0.80	0.43	0.00
tsp225	0.33	0.00	0.00

D. *Discussion*

The reason PSA/AT shows high performance for searching optimum solution is that the temperatures in PSA are determined adaptively so that they are included in the important temperature region for a given problem. This can be seen from Fig. 7, where the annealing temperatures in PSA/AT are shown as a function of the annealing steps for some TSPs.

The initial temperatures are determined randomly so that they spread over the very large range of temperature, but they converge quickly to a certain value, and fluctuate around it. This fluctuation is generated by the crossover and mutation in the GA operation, and the converged temperature is the important temperature for each problem. For the ch150 problem, the important temperature is around 9 from Table 1, and the evolving temperatures in PSA/AT also fluctuate around the same temperature. This is also true for the gil 62 and pr152 problems.

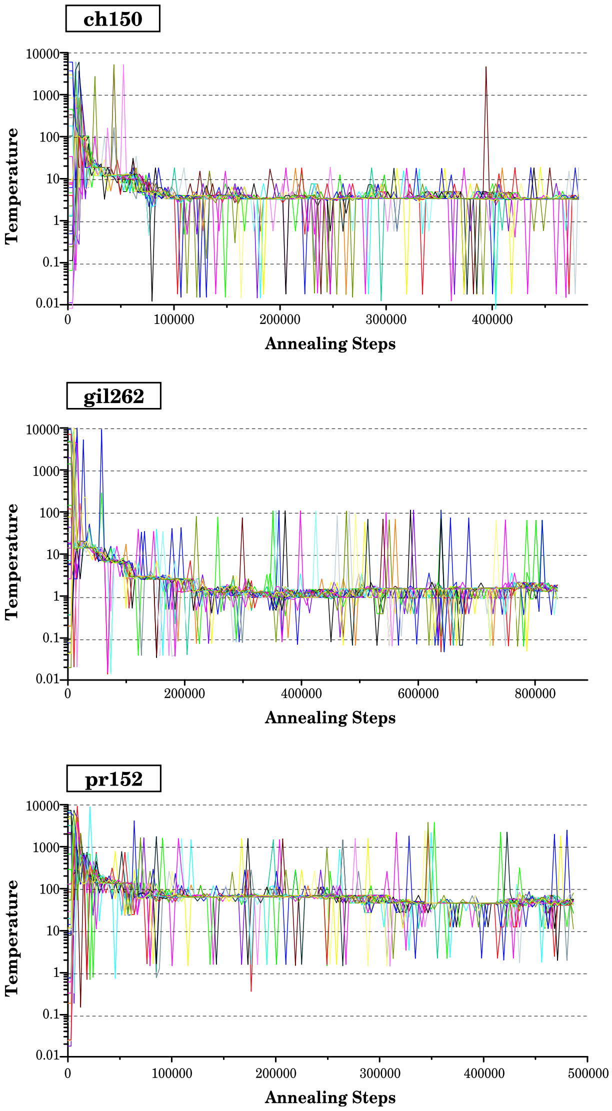

Fig. 7. Histories of temperatures in PSA/AT

Consequently, the proposed mechanism for determining the appropriate temperatures by GA is found very effective, and PSA/AT can be considered to be a useful parallel SA method.

It should be noted that the temperature adaptation mechanism adopted here can be realized with parallel SA since the criterion for selecting ggoodh temperatures can be established from the relative value and the relative fluctuation of the energies of the multiple SA processes. From this standpoint of view, parallelization of SA will give another new insight to optimization research field as well as speedup.

V. Conclusions

A new parallel simulated annealing method with adaptive temperature mechanism is proposed here. The conclusions are as follows.

1) It is not easy to determine an appropriate temperature schedule for a discrete optimization problem in simulated annealing (SA), but the SA with a specific constant temperature, which is called the important temperature here, can yield very good solutions.

2) The behavior of the transitions of solutions is investigated, and the characteristics of the behavior is found for the solutions at the important temperatures.

3) It is found that the temperatures of parallel SA processes can be optimized using GA and the above characteristics.

4) A new parallel SA with the above temperature adaptation mechanism is proposed, and the effectiveness and the usefulness of the proposed method are shown clearly for the Traveling Salesman Problems. This method is called PSA/AT and the method is very easy to use since we do not have to determine the temperature schedule, and it gives very good solutions as well.

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